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The entanglement of attitudes toward inequality: the theoretical background and measurement for the EU countries in 2021

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Abstract

This paper assumes two types of social planners who evaluate income distributions in terms of social welfare, economic inequality, and poverty. The first type, SP_ε, denotes individuals who have an aversion to income inequality as measured by the normative parameter ε. The second, SP_v, comprises individuals who have an aversion to rank inequality, as measured by the normative parameter v. Since every member of a society may play the role of a social planner, there could be as many levels of ε and v as there are society members. It raises the question of which ranges of v and ε values are ethically sensible when conducting empirical welfare studies. This paper proposes the answer to this question by introducing the concepts of inequality-entangled SP_v and SP_ε. If a randomly selected SP_v had v_i, one could automatically find ε_i of the inequality-entangled SP_ε, and vice versa. The inequality-entangled SP_v and SP_ε consistently evaluate inequality, social welfare, and poverty. This paper moreover proposes a method for eliciting the pairs (v_i, ε_i), $i = 1, 2, \ldots, n$, from empirical income distributions. Moreover, a single pair (v^*, ε^*) exists, representing all n pairs. Additionally, the study applies the inequality-entanglement methodological framework to assess social welfare, inequality and poverty for 27 European Union member countries in 2021.

Key words: income distribution, social welfare, inequality, poverty, inequality aversion, European Union.

1. Introduction

Applied welfare economics delegates the measurement of social welfare embodied in income distributions to an abstract *social planner* (*SP*) who uses individual *social evaluation functions* (*SEF*). Every member of society may play the role of the social planner with the same probability (Harsanyi, 1980).

This paper assumes two types of social planners: individuals who have an aversion to *income inequality* (SP_ε) and individuals who have an aversion to *rank inequality* (SP_v). An SP_ε uses Atkinson's (1970) index of income inequality $A(\varepsilon)$, where normative parameter $\varepsilon > 0$ reflects *aversion to income inequality.* The greater the value of ε, the

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more sensitive SP_ε is to *income differences.* An SP_v uses the extended (generalized) Gini index $G(v)$ (Yitzhaki, 1983; Kakwani, 1980; Donaldson and Weymark, 1980). The normative parameter $v \geq 1$ reflects an *aversion to rank inequality*. The greater the value of v, the more sensitive SP_v is to *rank differences,* regardless of the exact value that income may take at that rank (Duclos, 2000).

As every member of society may be a social planner with the same probability, there could be as many values of ε and v as society members. It raises the question of which ranges of v and ε values are ethically sensible when conducting empirical welfare studies (Duclos, 2000).

This paper proposes an answer to this question by introducing the concept of *entanglement of attitudes toward inequality* (hereafter, *inequality entanglement*). Let V and $\mathcal{E}$ be the sets of admissible levels of v and ε, respectively, and let $\boldsymbol{Vx\mathcal{E}}$ be the Cartesian product of V and $\mathcal{E}$. The economic theory allows for (resp. does not prohibit) the existence of pairs of SP_v and SP_ε (resp. v and ε) who consistently assess inequality in a given income distribution, namely that the following equality holds:

$$G(v) = A(\varepsilon) \tag{1}$$

for all pairs $(v, \varepsilon) \in \boldsymbol{Vx\mathcal{E}}$. We will call the pairs (SP_ε, SP_v), or (v, ε), satisfying Eq. (1), *inequality-entangled social planners.*

The concept of inequality-entangled social planners has various advantages. Such planners consistently assess income inequality. We will show that they also consistently assess social welfare and poverty in income distributions.

In applications, the concept of inequality-entangled social planners significantly narrows the range of v and ε values. We will show that there is a unique pair $(SP_{v^*}, SP_{\varepsilon^*})$, resp. (v^*, ε^*), representing all inequality-entangled pairs of social planners. We propose a method for estimating the pairs (v, ε) and (v^*, ε^*) from income data.

What are the rationales for applying quantum physics concepts? Orrell (2024) notices that neoclassical economics had roots in classical mechanics. The influence of mechanics persists in concepts such as the static equilibrium and the idea that people behave as independent, rational utility maximisers. However, economics shaped by uncertainty, dynamism and entanglement might be more applicable to the real world (Facco & Fracas, 2022). Section 2 explains the use of such a quantum physics metaphor in more detail.

The remainder of this paper is organized as follows. Section 2 introduces basic concepts and formulae. This Section also offers a brief literature review. Section 3 describes the method for estimating pairs (v, ε). Section 4 comprises the first part of the empirical results. After describing the EU-SILC income data, this Section presents estimates of the pairs (v^*, ε^*), social welfare, and economic inequality for 27 EU countries in 2021. Section 5 offers estimates of poverty. Section 6 concludes.

2. Quantum entanglement of particles and inequality entanglement of social planners

A phenomenon in which some social planners satisfy Eq. (1) resembles, metaphorically, the *quantum entanglement of particles*. Suppose two distinct quantum states, q_v or q_ε, characterize some subatomic particles. The quantum state of a particle is unknown before measurement.

Quantum entanglement is the phenomenon of a system of particles such that the measurement of one particle's quantum state, say q_v, *automatically* provides the measurement of its companion's state, say q_ε, even when a vast distance separates the particles.

An example of entanglement is a subatomic particle that decays into an entangled pair of other particles. The decay events obey the various conservation laws. As a result, the measurement outcomes of one particle must be highly correlated with the measurement outcomes of its companion particle, whereas the total momenta, angular momenta, energy, or the like remain the same before and after this process (Caltech Science Exchange 2024). Many entangled particles may exist.

Regarding social planners, note that if a randomly selected person is to play the role of a social planner, we do not know in advance whether they are SP_ε or SP_v. *Inequality entanglement of the attitudes toward inequality* (*inequality entanglement*, for short) is a metaphor for the phenomenon of a group of social planners, so that just a measurement of v (resp. ε) *automatically* gives the measurement of ε (resp. v) due to Eq. (1). There may exist a multitude of inequality entangled pairs SP_v and SP_ε.

If a person becomes an actual social planner, she should provide an unambiguous assessment of inequality in the analyzed income distribution. Suppose she announces: "$G(v)$ equals 0.32". It reveals that she is an SP_v with an aversion to rank inequality equal to $v = G^{-1}$ (0.32). Then her *inequality-entangled* companion, SP_ε, should have $\varepsilon = A^{-1}(0.32)$. On the other hand, if the selected person's answer was: "$A(\varepsilon) = 0.32$", an entangled person should have $v=G^{-1}(0.32)$. There may exist a multitude of *inequality-entangled SP_v* and SP_ε.

A change from $G(v)$ into $A(\varepsilon)$ (and vice versa) *preserves income inequality*. In other words, Eq. (1) plays the role of a *conservation law*. Moreover, such a change also preserves social welfare and poverty. To see this, note that for an analyzed income distribution with $\mu > 0$, Eq. (1) is equivalent to:

$$\mu[1\text{-}G(v)] = \mu[1-A(\varepsilon)]. \tag{2}$$

In Eq. (2), $\mu[1–G(v)]$ and $\mu[1–A(\varepsilon)]$ are the *Social Evaluation Functions*, SEF_ε and SEF_v implied by $G(v)$ and $A(\varepsilon)$, respectively. Thus, *inequality-entangled* social planners consistently assess social welfare. It is worth adding that $\mu[1\text{-}G(v)]$ and $\mu[1–A(\varepsilon)]$ in Eq. (2) are the *Equally Distributed Equivalent Incomes (EDEI),* which, if received by all persons, give the same level of *social welfare* as the present distribution (Atkinson, 1970).

Eq. (1) also implies that inequality-entangled social planners consistently assess poverty in income distributions. A person is deemed poor if his/her income is less than a normative *poverty line z* established by a social planner. Kot and Paradowski (2024a) argue that the *EDEI* is an *upper limit* of any socially acceptable poverty line *z*, namely,

$$z \leq EDEI \tag{3}$$

If a social planner proposed a poverty line *z* greater than *EDEI*, attaining an egalitarian income distribution would be possible at the cost of common poverty. Arguably, no reasonable society would accept such a poverty line. Kot and Paradowski (2024) refer to such a peculiar situation as the *Equity-Poverty Trap*.

Note that Eq. (2) expresses the equality of poverty lines and, therefore, the equality of poverty indices, which are monotonic functions of a poverty line. Thus, inequality-entangled social planners consistently assess poverty in a given income distribution.

In this paper, we will use the following family of poverty indices:

$$FGT_\alpha = \sum_{x_i<z} \left(\frac{z-x_i}{z}\right)^\alpha p_i, \tag{4}$$

where z is the poverty line, x_i is income below z, and α is a normative parameter (Foster, Greer, and Thorbecke, 1984).

Some particular cases of Eq. (4), namely FGT_0, FGT_1, and FGT_2, are widely used. FGT_0, also known as the head-count *ratio*, measures poverty incidence. FGT_1, called the *poverty depth*, measures the poverty of society as a whole (Foster and Shorrocks, 1991). FGT_2 measures *poverty severity.*

3. Parametric utility functions and social evaluation functions

3.1. Personal and moral preferences

According to Harsanyi (1980, pp. IX-X), each individual has two kinds of preferences. The first kind comprises his *personal preferences*, which are defined as his actual preferences based on his interests. The second one consists of *moral* preferences defined as a person's "(…) *hypothetical* preferences that he *would* entertain if he forced himself to judge the world from a moral, i.e. from an impersonal and impartial point of view". More specifically, "(…) *moral* preferences are those preferences that he would entertain if he assumed to have the same probability $1/n$ to be put in place of any one of the n individual members of society" (Harsanyi, 1980, pp. IX). Mathematically, an individual's personal preferences are represented by his *utility function,* whereas his *social evaluation function* represents his moral preferences.

Concerning moral preferences, a rational individual would try to maximize his expected utility, thereby maximizing the average utility of the individual members of society. It means that a rational individual will always use the average utility level in society as his social evaluation function.

Harsanyi (1980, p. X) noticed that this definition of social evaluation functions presupposes the possibility of *interpersonal comparisons* of utility. He argued that "(…) interpersonal utility comparisons are essentially the same kind of mental operation as intrapersonal utility comparisons are."

3.2. The social evaluation function of averters to income inequality

In this paper, we will use the following terms and symbols. The positive valued random variable X, with the distribution function $F(x) = \mathrm{P}(X \leq x)$, will describe the distribution of personal incomes. We assume that the mean $\mu = \mathrm{E_F}[\mathrm{X}]$ exists and is finite, where the operator $\mathrm{E_F}[\cdot]$ is the mathematical expectation of X with respect to $F(x)$.

We assume that an averter to income inequality uses the utility function of the form:

$$u(x) = \begin{cases} \frac{\mathrm{x}^{1-\varepsilon}}{1-\varepsilon}, for\ \varepsilon \neq 1 \\ ln\mathrm{x}, for\ \varepsilon = 1 \end{cases}, x>0, \tag{5}$$

(Atkinson, 1970). Eq. (5) defines the *utility function of constant relative inequality aversion* (*CRIA*).

For averters to income inequality, the Social Evaluation Function is the expected value of $u(X)$ with respect to the distribution F, namely:

$$SEF_\varepsilon = E_F[u(X)] = \begin{cases} \frac{E_F[\mathrm{X}^{1-\varepsilon}]}{1-\varepsilon}, for\ \varepsilon \neq 1 \\ E_F[ln\mathrm{X}], for\ \varepsilon = 1 \end{cases}, \tag{6}$$

(Atkinson, 1970).

The parameter ε measures *a social planner's or society's aversion to income inequality*. When $\varepsilon < 0$, a social planner or society is *averse to equality*. Null inequality aversion, i.e. $\varepsilon = 0$, characterizes an *inequality-neutral* society. In this case, $SWF_0 = \mu$, *and value judgments about income distributions are based only on mean incomes, providing no information about* income inequality. Thus, income distribution X with the mean μ_x is preferred over Y with the mean μ_y if and only if $\mu_x > \mu_y$. If $\varepsilon > 0$, *society is inequality-averse*. Hereafter, we will assume $\varepsilon \geq 0$.

Knowledge of ε is essential for various reasons. As ε ultimately determines the function (5), it enables a direct measurement of SEF_ε. Parameter ε expresses the rate at which a society solves the trade-off between efficiency and equality. As the (minus) elasticity of the marginal utility of income, ε, also has a central role in public Economics: high values of ε mean that the marginal utility of income declines as income grows, and therefore, an income transfer from the rich to the poor is increasingly desirable (Young, 1990). Knowledge of ε is also essential for appraising social projects and policies that impact different socioeconomic groups (Evans, 2005; Layard et al., 2008; Aristei and Perugini, 2016).

Based on the interpretation of ε as inequality aversion, Atkinson (1970) proposed the normative index of inequality:

$$A(\varepsilon) = \frac{\mu-\mu_\varepsilon}{\mu}, \tag{7}$$

where μ_ε is *the equally distributed equivalent income* (*EDEI*) that, if distributed equally, gives the value of the social evaluation function $\mathrm{E}[u(X)]$ the same as the initial distribution (Kolm, 1969; Atkinson, 1970; Sen, 1973). In general, *EDEI* is the solution to the equation: $u(EDEI) = \mathrm{E}[u(X)]$ for a given utility function $u(x)$.

For the utility function (5) and social welfare function (6), *EDEI* has the following form:

$$\mu_\varepsilon = \begin{cases} \{\mathrm{E}[u(X)]\}^{1/(1-\varepsilon)}, & for\ \varepsilon \neq 1 \\ \exp\{E[lnX]\}, & for\ \varepsilon = 1 \end{cases}. \tag{8}$$

For ε=1, μ_ε is the geometric mean, and for $\varepsilon = 2$, μ_ε is the harmonic mean. For a given income distribution, μ_ε is a declining function of ε (Lambert, 2001, Chapter 4).

It follows from (7) and (8) that *EDEI* is a money metric of the social evaluation function *SEF,* namely:

$$\mu_\varepsilon = \mu[1-A(\varepsilon)], \tag{9}$$

(Atkinson, 1970). Eq. (9) specifies the family $\{SEF_\varepsilon\}_{\varepsilon\geq 0}$ of social evaluation functions indexed by ε.

3.3. The social evaluation function of averters to rank inequality

Sen (1973, p. 41) argued that the social value of the welfare of individuals should depend crucially on the levels of welfare (or incomes) of others. The following social evaluation function satisfies this claim:

$$\mu_v = \mu[1-G(v)], \tag{10}$$

where $G(v)$ is the extended Gini index of the form:

$$G(v) = 1 - v(v-1)\int_0^1 (1-p)^{v-2} L(p)dp, v \geq 1, p \in [0,1], \tag{11}$$

(Yitzhaki, 1983; Donaldson and Weymark, 1980; Kakwani, 1980). In Eq. (11), *L(p)* is the Lorenz curve, $1-p = 1-F(x)$ is the rank of a person with income *x*, and v is a normative parameter expressing *aversion to rank inequality.* The case $0 \leq v < 1$ reflects *rank equality aversion,* $v = 1$ *rank equality neutral*, and $v \geq 1$ *rank inequality aversion* (Yitzhaki, 1983; Duclos, 2000). For $v = 2$, $G(v)$ is the ordinary Gini index.

Eq. (10) defines the family $\{SWF_v\}_{v>1}$ of social welfare functions indexed by $v > 1$. More specifically:

$$\mu[1 - G(v)] = v \int_0^{\infty} x[1 - F(x)]^{v-1} f(x)dx, \tag{12}$$

(Lambert, 2001, p. 125).

Yitzhaki (1983) noted that $G(v)$ (11) has most of the properties of Atkinson's index (7). Indeed, at the extremes $v \to 1$ and $v \to \infty$, the behavior of $G(v)$ resembles that of the $A(\varepsilon)$ at the extremes $\varepsilon \to 0$ and $\varepsilon \to \infty$ of inequality aversion (Lambert, 2001, p. 115). As $v \to 1$, $G(v) \to 0$. As $v \to \infty$, $G(v) \to 1-L'(0)$. For a discrete distribution of *X*, $G(v) \to 1-x_{min}/\mu$ as $v \to \infty$.

4. Estimating aversion to income inequality and rank inequality.

4.1. The previous methods of estimating *ε* and *v*

The literature offers various methods of recovering ε from empirical data. Inequality aversion ε has been elicited from Okun's "leaky bucket" experiment (Okun, 1975). In this experiment, participants subjectively assess a tolerable 'leakage' of money due to administrative costs during income transfers among individuals. The higher the leakage the participants permit, the greater their aversion to income inequality.

One can elicit ε from *the equal sacrifice model* (Richter, 1983; Vitaliano, 1977; Young, 1987). Lambert et al. (2003) elicit ε by hypothesizing about *the natural rate of subjective inequality.* Kot (2020) proposes the estimator of $\hat{\varepsilon} = (ap + 1)/2$ when income obeys the Generalised Beta distribution of the second kind GB2(*x*; *a, b, p, q*) (McDonald, 1984).

Much less is known concerning the range of v that analysts may apply in empirical studies. Kot (2022) analyses the empirical relationship between the generalized Gini index $G(v)$ and three Italian indices of inequality, namely the Pietra (1915) index, the Bonferroni (1930) index, and the Zenga (2007) index. The author finds these indices corresponding to $G(v)$ with v equal to 1.5, 3, and 11, respectively.

Duclos (2000) recommends the leaky bucket experiment for deriving v. The author argues that v should not exceed 4 in empirical analyses if the whole transfer is not licked. In this paper, we will follow Duclos' recommendation for an upper limit of v.

The joint estimation of ε and v has not yet been analyzed, with one exception. Recently, Kot and Paradowski (2024b) obtained the pairs (ε^*, v^*) for ten Latin American and Caribbean countries by solving the system of two nonlinear equations: $SWF_\varepsilon = SWF_v$ and $x_\varepsilon^* = x_v^*$, where x_ε^* and x_v^* are the *benchmark incomes* of SP_ε and SP_v, respectively. The authors acknowledged that their method requires further improvement.

4.2. The mean-value method

We propose a three-stage method for estimating the pairs (v^*, ε^*). In the first stage, we generate *n* random values of aversion to rank inequality $v_1, v_2, \ldots, v_n$ from the uniform distribution U[1,4] and estimate the sequence of *n* extended Gini indices, $G(v_1), \ldots, G(v_n)$ for an analyzed income distribution.

To justify this stage, note that the state of complete ignorance concerning the value of *v* means the state of *maximum entropy*. The uniform distribution has the maximum entropy among all probability distributions defined on finite intervals (Cover and Thomas, 1991, p. 269). Thus, one may expect *v* to follow a uniform distribution U[1,4].

In the second stage, we calculate *n* values of ε_i as the solutions to Eq. (1) for $G(v_i)$. Thus, we get n possible pairs (v_i, ε_i) of inequality-entangled social planners. We refer to the graph of the pairs as the *v-ε curve* or the *ε(v)* function. Fig. 1 illustrates the *v-ε* curves for some European countries.

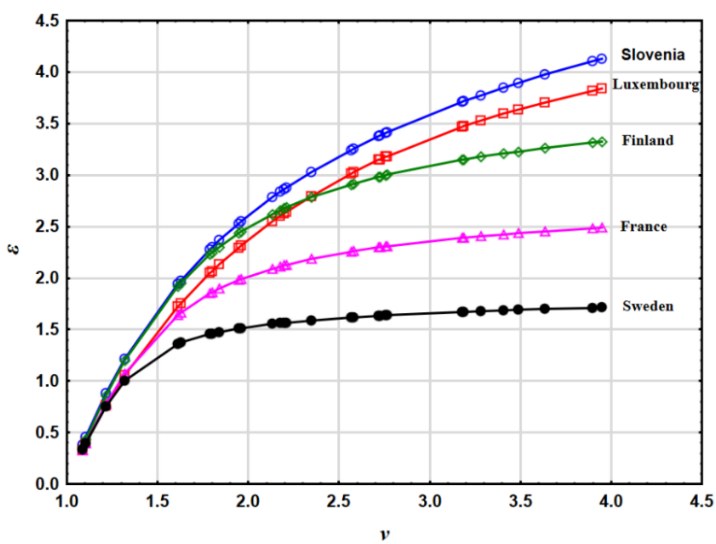

Figure 1. The *v-ε* curves for selected UE countries in 2021

Source: own work.

Every point in Fig. 1 represents a combination of *exogenous v* and corresponding ε, guaranteeing the same inequality assessment in a given income distribution by a pair of inequality-entangled social planners. At the upper limit of $v = 4$, the curves in Fig. 1 attain different levels, which depend on the country's income distribution.

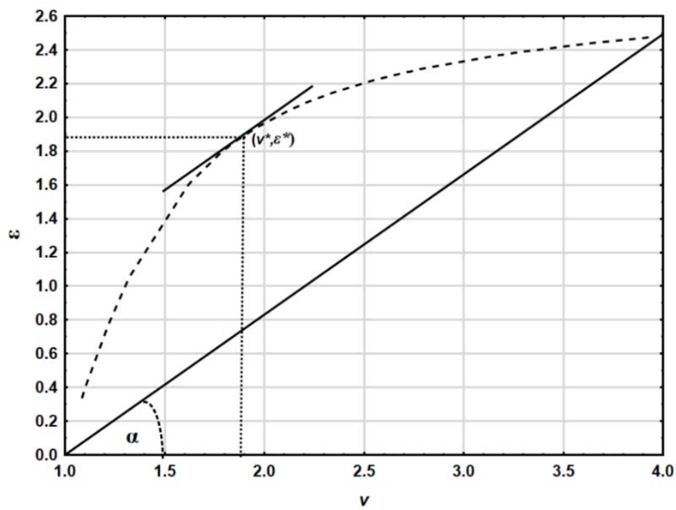

Figure 2. A single representation (v^*, ε^*) of the v-ε curve for Poland, 2021

Source: *own work.*

In the third stage, we search for a unique pair (v^*, ε^*) representing all inequality-entangled pairs (v_i, ε_i). The v-ε curve in Fig. 2 illustrates the concept of this stage.

In Fig. 2, the tangent of angle α reflects *the average proportion* of ε to v that gives the same inequality assessment. More specifically:

$$\tan(\alpha) = \frac{\varepsilon(v_{max}) - \varepsilon(v_{min})}{v_{max} - v_{min}}, \tag{13}$$

where the $v_{min} \geq 1$, and $v_{max} = 4$. The corresponding $\varepsilon(v_{min})$ and $\varepsilon(v_{max})$ are calculated from Eq. (1).

If the function $\varepsilon(v)$ is differentiable within the interval $[v_{min}, v_{max}]$, then Lagrange's mean value theorem implies the existence of a point v^* inside this interval at which the following equation holds:

$$tan(\alpha) = \varepsilon'(v^*), \tag{14}$$

where $\varepsilon'(v^*)$ is the first derivative of $\varepsilon(v)$ at v^*.

In Fig. 2, the point (v^*, ε^*) reflects the *average level of income inequality aversion* for offsetting rank inequality aversion when attaining the same inequality assessment in a given income distribution. In this sense, the single point (v^*, ε^*) represents all points on the v-ε curve.

We propose to calculate the derivatives $\varepsilon'(v_i)$ analytically from a function fitted to the empirical v-ε curves. It has turned out that the following function approximates the v-ε curves "almost ideally":

$$\varepsilon(v) = \theta_1 + \theta_2 v - \exp\{\theta_3 - \theta_4 v\}, \tag{15}$$

where the parameters $\theta_1, \theta_2, \theta_3$ and θ_4 can be estimated using the nonlinear least squares method.

We use the ratio of the regression sum of squares to the total sum of squares as a measure of goodness-of-fit. As this ratio explains the proportion of variance accounted for in the dependent variable (ε) by the model (15), this is equivalent to the coefficient of determination R^2 ($0 \leq R^2 \leq 1$).

Equating the derivative of (15) with *tan(α)* gives:

$$\tan(\alpha) = \theta_2 + \theta_4 \exp\{\theta_3 - \theta_4 v^*\}. \tag{16}$$

After simple algebra, we get:

$$v^* = \left\{\theta_3 - \log\left[\frac{(tan(\alpha) - \theta_2)}{\theta_4}\right]\right\}/\theta_4. \tag{17}$$

The corresponding parameter ε^* can be calculated from Eq. (15).

By substituting the sample estimates for the parameters θ_2, θ_3, and θ_4 in equation (17), we obtain the estimator of v^*. We shall refer to this way of obtaining (v^*, ε^*) as *the mean-value method* (MVM).

5. Empirical results for the EU-member countries 2021

5.1. Statistical data

We use statistical data on household disposable income [in Euros] from the EU-SILC database for 2021. To obtain a distribution of personal disposable incomes, we adjust household incomes by the square-root equivalence scale (Buhmann et al., 1988). Such an adjustment requires weighing the resulting equivalent incomes. We follow the common practice of weighting adjusted incomes by household size. The final weights applied in this paper are products of household size and cross-sectional survey weights.

We generate the sequence of 30 random values of v_i from the uniform distribution U[1,4]. Then we estimate the sequence of $G(v_i)$, $i = 1, \ldots, 30$, for every country and the corresponding sequence of ε_i, solving Eq. (1) numerically using the IMSL Fortran subroutine NEQNF. Next, we estimate the parameters θ_1, θ_2, θ_3 and θ_4 of the nonlinear function (15) for every country using the pairs (ε_i, v_i), i=1,…, 30 and the IMSL Fortran subroutine RNLIN.

5.2. Estimates of v* and ε*

Table 1 presents the results of applying the MVM to EU-SILC data. Besides estimates of ε* and ν*, this Table contains the estimates of the generalized Gini index *G(ν*)* (equal to the Atkinson index *A(ε*)*) and *EDEI*. The last column of this Table (labelled as R^2) contains the values of the coefficient of determination. For further comparison, this Table also includes the mean equivalent income. The last row of this Table (labelled 'EU total') comprises results for all EU member countries' incomes and weights.

Table 1. Estimates of income inequality *G(ν*) = A(ν*)* and social welfare, *EDEI* [€] based on the inequality-entangled estimates of aversion to rank inequality ν* and income inequality ε* for EU-member countries in 2021

No.	Country	*ν**	*ε**	*GA*	*EDEI* [€]	Mean_D [€]]	R^2
1	Austria	1.71847	1.30212	0.22367	25637	33023	0.9997
2	Belgium	1.88000	2.05323	0.22847	23435	30375	0.9999
3	Bulgaria	2.06051	1.95660	0.40729	4463	7530	1.0000
4	Croatia	1.94088	1.76289	0.28500	7306	10218	1.0000
5	Cyprus	1.94303	2.21251	0.28839	15711	22078	0.9997
6	Czechia	1.99332	2.52619	0.25467	9854	13221	1.0000
7	Denmark	1.78069	1.68096	0.24039	29321	38600	0.9998
8	Estonia	1.90745	1.67218	0.29094	11044	15576	1.0000
9	Finland	1.97727	2.46902	0.26103	22752	30789	1.0000
10	France	1.87692	1.92728	0.27535	20694	28557	0.9999
11	Germany	1.87262	1.79843	0.28716	22548	31631	0.9997
12	Greece	1.89762	1.66411	0.29579	7975	11325	0.9999
13	Hungary	1.86140	1.75326	0.25622	6064	8153	0.9999
14	Ireland	1.75840	1.67332	0.23879	27216	35754	0.9997
15	Italy	1.79748	1.38777	0.28947	16030	22561	0.9999
16	Latvia	1.93202	1.60822	0.34519	8153	12451	1.0000
17	Lithuania	1.99808	1.84871	0.35679	8346	12976	1.0000
18	Luxembourg	2.11278	2.52324	0.30744	36250	52342	1.0000
19	Malta	1.66892	1.31318	0.24643	16123	21395	0.9994
20	Netherlands	1.84791	1.87964	0.25099	19377	25870	0.9999
21	Poland	1.89022	1.88695	0.24884	7944	10576	1.0000
22	Portugal	1.93503	1.72011	0.31758	9978	14621	1.0000
23	Romania	1.95258	1.53072	0.32642	4197	6231	0.9999
24	Slovakia	1.92978	2.10286	0.21011	8210	10394	0.9999
25	Slovenia	2.02849	2.64598	0.24319	14044	18557	0.9999
26	Spain	1.84606	1.41535	0.29774	14259	20304	0.9999
27	Sweden	1.70576	1.41729	0.22131	23228	29830	0.9997
	EU total	**1.90295**	**1.52573**	**0.32298**	**16153**	**23859**	**1.0000**

Note: R^2 measures the goodness-of-fit of the model (15).
Symbol *GA* denotes the common level of inequality *G(ν*) = A(ε*); Mean_D* is the mean equivalent income.

Source: own calculations using EU-SILC data.

Inspecting R^2 in Table 1 shows an almost ideal fitting of the ε-v curves by function (15). Moreover, normative parameters v and ε are country-specific. For instance, Spanish and Dutch social planners have a similar aversion to rank inequality, $v \approx 1.85$. However, Spanish inequality-entangled SP_ε should have $\varepsilon \approx 1.42$ to assess income inequality identically as his companion SP_v did. On the other hand, the Dutch inequality-entangled SP_ε must have $\varepsilon \approx 1.88$ for the same purpose. Similarly, the last row of Table 1 indicates that a European SP_v with $v = 1.90295$ assesses social welfare (*EDEI*) in the EU as €16,153. His inequality-entangled companion SP_ε provides the same welfare assessment when $\varepsilon = 1.52573$**.**

Table 2 provides additional information on the distributions of the characteristics presented in Table 1.

Table 2. Descriptive statistics of estimates in Table 1

Parameter	Mean	Median	Min.	Max.	Std. Dev.	V [%]	Skew-ness	Ku
v^*	1.89310	1.89762	1.66892	2.11278	0.10703	5.65	-0.237	0.047
ε^*	1.84193	1.76289	1.30212	2.64598	0.37636	20.43	0.674	0.191
GA	0.27758	0.27535	0.21011	0.40729	0.04588	16.53	0.949	1.059
EDEI	15561	14259	4197	36250	8532	54.83	0.611	0.422
Mean D	21293	20304	6230	52342	11404	53.56	0.787	0.357

Note: LB and *UB* are the lower and upper limits of the 95% confidence interval; *V* is the coefficient of variability; *Skew* is the coefficient of skewness; *Ku* is the kurtosis.

Source: own calculations using data from Table 1.

Examining Table 2 shows that v^* is statistically significantly less than 2.0 at a 0.05 significance level (*p-value* = 00001). Thus, analyzing income inequality using the standard Gini index, *G(2),* is debatable. Variability of this parameter is slightly lower than that of ε^*. Moreover, the distribution of v^* across countries is negatively skewed, whereas the distribution of ε^* is positively skewed. Both distributions are flatter than the standard normal distribution.

Note that, in Table 2, the parameter means differ from those for "EU-total" in Table 1, except for v^*. It might be because we calculated descriptive statistics of the parameter distributions across countries without weighting. The discussed differences are more evident in the case of inequality measures, since they are not additively decomposable.

5.3. Economic poverty in EU Member Countries in 2020

As mentioned in Section 2, inequality-entangled social planners consistently assess poverty in an income distribution. When we set a country's *EDEI* as a national poverty line z, the FGT_α indices (4) enable assessments of various aspects of the country's impoverishment.

EDEI, as an upper limit of poverty lines, inherently implies an *international poverty line*. If there were rationales for comparisons of poverty across *N* selected countries, an international poverty line, $z_{all,}$ should satisfy the following condition:

$$z_{int} = \min_{i}\{z_1, z_2, \dots, z_N\}, i = 1, 2, \dots, N \tag{18}$$

where $z_1, \dots, z_N$ are the country's poverty lines equal to the countries' *EDEI*s. (Kot, Paradowski, 2024a). The international poverty line, as defined by (18), guarantees that no selected country falls into the *Equity-Poverty Trap*.

Table 3 presents estimates of the *FGTα* poverty indices (12) for α = 0, 1, and 2.

Table 3. Poverty in EU Member Countries in 2020.

No.	Country	***National poverty lines***			***International poverty line***		
		z_i= $EDEI_i$ *in Table 1*			z_{int}=4197 (Romania)		
		FGT_0	FGT_1	$FGT2_2$	FGT_0	FGT_1	$FGT2_2$
1	Austria	0.36581	0.10582	0.04873	0.01143	0.00787	0.00659
2	Belgium	0.34246	0.08842	0.03331	0.00386	0.00175	0.00105
3	Bulgaria	0.35380	0.11823	0.05542	0.31664	0.10431	0.04810
4	Croatia	0.33375	0.11037	0.05319	0.10984	0.03284	0.01533
5	Cyprus	0.36184	0.09453	0.03514	0.00255	0.00095	0.00058
6	Czechia	0.32988	0.07594	0.02769	0.01674	0.00413	0.00167
7	Denmark	0.36599	0.09620	0.03921	0.00443	0.00205	0.00142
8	Estonia	0.36033	0.11800	0.05462	0.03432	0.01215	0.00677
9	Finland	0.33782	0.08149	0.02879	0.00134	0.00045	0.00023
10	France	0.35090	0.09391	0.03790	0.00535	0.00244	0.00147
11	Germany	0.35968	0.10394	0.04514	0.00580	0.00197	0.00111
12	Greece	0.34707	0.11060	0.05256	0.08125	0.02651	0.01396
13	Hungary	0.34633	0.09805	0.04322	0.13057	0.03774	0.01787
14	Ireland	0.37738	0.10011	0.03874	0.00190	0.00145	0.00122
15	Italy	0.35964	0.12254	0.06170	0.02593	0.01086	0.00699
16	Latvia	0.35922	0.12914	0.06469	0.10611	0.03147	0.01564
17	Lithuania	0.35290	0.11525	0.05370	0.07271	0.02273	0.01138
18	Luxembourg	0.32825	0.09153	0.03611	0.00003	0.00002	0.00001
19	Malta	0.40029	0.12068	0.05332	0.01124	0.00647	0.00452
20	Netherlands	0.36097	0.09178	0.03624	0.00701	0.00283	0.00168
21	Poland	0.33929	0.09580	0.04091	0.06027	0.01658	0.00779
22	Portugal	0.34087	0.11130	0.05403	0.05156	0.01785	0.00974
23	Romania	0.33552	0.13102	0.07162	0.33552	0.13102	0.07162
24	Slovakia	0.31707	0.08255	0.03454	0.04662	0.01368	0.00636
25	Slovenia	0.31133	0.07859	0.02956	0.00386	0.00084	0.00029
26	Spain	0.35373	0.12800	0.06751	0.03944	0.01670	0.01030
27	Sweden	0.37262	0.10814	0.04663	0.00828	0.00425	0.00297
1-27	**EU total**	**0.34471**	**0.12646**	**0.06723**			

Source: own calculations using data from the EU-SILC database.

On the left panel of Table 3, one can see high poverty levels in all countries. It is worth noting that *EDEI*, as an upper limit of poverty lines, implies upper limits for poverty measures. The differences between countries' FGT_0, FGT_1, and FGT_2 estimates are relatively small. Thus, according to national poverty standards, all analyzed countries might be expected to have similar poverty incidence, depth, and severity. However, these results are only for *internal use* and do not support international comparisons.

When we apply Romania's *EDEI* of €4197 as the international poverty line (according to Eq. 18), the right panel of Table 2 shows a much greater diversity of poverty assessments across EU Member Countries than the left panel. Bulgaria and Romania are among the poorest countries, as measured by three poverty indices. On the other end of the spectrum, Luxembourg is the most affluent country.

The inequality entanglement is a conceptual novelty of this paper. It enables the elicitation of normative parameters v and ε from empirical income distributions. Knowledge of these parameters enables consistent assessments of inequality by the social planners SPv and $SP\varepsilon$, who employ different methodologies. The inequality entanglement also enables consistent assessments of social welfare and economic poverty.

6. Concluding remarks

The method of eliciting pairs (v, ε) from income data may start with generating random numbers of ε instead of v from U[1,4] distribution. If incomes obey the generalized beta distribution of the second kind GB2(a,b,p,q), Kot (2020) demonstrates that ε belongs to the [0, ap+1] interval. Therefore, one can generate n non-random numbers from the U[0, ap+1] distribution and then calculate the inequality-entangled v using Eq. (1).

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